Unit of Focus: Quivers of Finite Representation Type

For my course "Mathematical Quivers", a course designed to introduce the basics of quiver algebras and representation theory to advanced undergraduate students, my unit of focus is Unit 2: Quivers of Finite Representation Type.

Introduction:

After learning the basics of what quivers and their representations are in the first unit, a natural question to ask is when do quivers have only finitely many representations, and if so, how can one come up with a comprehensive list of these representations? Such questions tie into the notion of the Auslander-Reiten quiver, or the AR-quiver, attached to any quiver. In this unit, we study these AR-quivers and discuss what information these quivers provide for an arbitrary quiver, namely its indecomposable representations and irreducible maps between them. In this unit, we also study Dynkin classifications of quivers and use Gabriel's theorem to tell us precisely when a quiver is of finite representation type. Finally, we will discuss how to actually construct the AR-quivers for certain classes of quivers (namely Dynkin type A_n and D_n). This discussion will prepare us for Unit 4 in which we will compare quivers with relations to those without when discussing finite representation type.

Sample learning objectives:

Students will be able to

- Given the AR-quiver of a quiver of finite representation type, determine the following for said quiver: indecomposable representations, projective indecomposable representations, injective indecomposable representations, simple representations. Describe *all* of the quiver's representations in terms of the indecomposable representations.
- Given the AR-quiver of a quiver of finite representation type, describe the irreducible morphisms between two indecomposable representations. Explain what information these irreducible morphisms tell us about the structure of the original quiver.
- Given the AR-quiver of a quiver of finite representation type, construct one possibility for the original quiver.
- Identify the Dynkin type of any of the following types of quivers: A_n, B_n, C_n, D_n, E_n, F_4, G_4
- Construct the AR-quiver of any type A_n or D_n quiver using the knitting algorithms.
- Determine whether or not a quiver has finite representation type without constructing the AR-quiver (using Gabriel's theorem).

- Construct a quiver of *infinite* representation type and prove said quiver has infinite representation type.

Assessment:

Assessment in this unit will be similar to that in Units 1 and 3:

- **Informal Assessment**: During most class periods, students will be given short problems to think about or complete. They will be able to discuss these questions in small groups, following which we will discuss the problems as a class. These will allow the instructor to determine student comprehension for recent materials and serve as an indicator of readiness for the next topic. These will not be graded.

- **Homework Sets**: Excepting the week of the exam, students will be given weekly homework sets to complete for credit. Each homework will consist of several problems, including computations and proofs similar to those seen in class.

- **Midterm**: The second midterm of the course will be over this unit. Students will have both an in-class and take-home exam to complete. The in-class exam will come first and will consist of short computations and short answer type questions (eg: Question #1 from below). The take-home exam will be given out immediately following the in-class exam. Students will have five days, during which they may choose any 48-hour block of time to complete the exam. Questions on the take-home exam will be more lengthy or difficult and may include proofs of results and longer computations similar to those from homework sets and from class discussion (eg: Question #2 from below).

Sample questions:

1. List *all* unbound quivers of finite representation type according to their Dynkin types. Draw one example of each of these different Dynkin types, making sure to label these quivers. Which named result provides this list of quivers? (5 *pts*)

Rubric: __/ 1 point: Students cite the appropriate theorem (Gabriel's theorem).

__/ 1 points: Students correctly provide the list of quivers (A_n, D_n, E_n)

__/ 3 points: Students are able to correctly draw three different quivers (1 point per quiver) that have finite representation type. For each quiver, students correctly identify its Dynkin type.

2. Calculate the AR-quiver of the following Dynkin type D_6 quiver. (*10 pts*)

(I would draw out the quiver on an actual assignment.)

Rubric: __/ 1 point: Students begin with each of the projective indecomposable modules.

__/ 1 point: Students end with each of the injective indecomposable modules.

__/ 3 points: Students correctly display the use of each of the three mesh techniques at least once.

__/ 4 points: Students find all intermediate indecomposable modules.

__/ 1 point: Students correctly identify the D_6 quiver.

Instructional Methods:

Lecture with discussion will be the primary instructional method used in teaching this unit as well as the other units. As most of the outside references for quivers will be very difficult for students to understand without years of graduate study, lecture is useful to present the material at a level appropriate for advanced undergraduate students. Discussion will also be used along the way so that students can both ask and answer question related to these challenging concepts in order to keep students engaged during class. Small group work and class discussion will also be used as an informal assessment technique most classes as described above so that students can immediately test their understanding of what is discussed in class.

Finally, specific to this unit, class presentations will be used when learning how to compute the AR-quivers of type A_n and D_n quivers. These will be used in this specific instance by having students go through and draw the AR-quivers on the board in front of other students as another form of informal assessment. This will help students who are struggling to see how the different mesh algorithms are carried out and will give students some practice in presentations before their larger-scale presentations at the end of the course.