SYLLABUS - FALL 2015

The University of Iowa
The College of Liberal Arts and Sciences
Department of Mathematics
14 MacLean Hall (MLH)

MATH:3900:0AAA Mathematical Quivers

Class location: 105 MLH
Meeting time: 11:30-12:20 PM, MWF

Some of the policies relating to this course (such as the drop deadline) are governed by its administrative home, the College of Liberal Arts and Sciences, 120 Schaeffer Hall.

Instructor: Kevin Gerstle
Office: MLH 325C
E-mail: kevin-gerstle@uiowa.edu
Office hours: MW: 1:00-2:00 PM,
Th: 10:30-11:30 AM, and by appointment

DEO Contact Information: Dan Anderson, 14 MLH

Description of Course:
This course is designed to introduce advanced undergraduate students to the representation theory of finite-dimensional algebras using the language of quivers. Topics discussed will include quivers, representation types, quiver morphisms, quivers of finite representation type, Gabriel's theorem, Dynkin classification, Auslander-Reiten quivers, ring/module theory, algebras, path algebras, bound quivers, and other topics as time allows.

Course Goals:
By taking this course, students will become familiar with one burgeoning area of research in modern mathematics. Students will learn what types of questions algebraists study and many of the techniques that are used to solve problems from representation theory and will learn how the tools developed in linear algebra and the concrete structures of quivers can be used to understand abstract spaces. This course is designed to prepare students for future in mathematics by helping students to develop proficiencies in critical thinking, problem-solving, and presentations.

Approved GE: Quantitative or Formal Reasoning

Requirements:
Grade of C+ or higher in both MATH:3720 (Introduction to Abstract Algebra I) and MATH:2700 (Introduction to Linear Algebra) or equivalent.

Required Materials:
1. Schiffler, R. (2014). *Quiver Representations*. Springer International Publishing.

This textbook provides an introduction to the study of quivers that is designed for advanced mathematics students. By carefully choosing topics from this text, we will use it as an appropriate source of information to supplement classroom discussion. Homework questions will be taken in part from this book.

ICON (https://icon.uiowa.edu/) and E-mail:
Class announcements, due dates, and information pertaining to homework, quizzes, and tests will be posted regularly on ICON. I expect students to check the course page daily.

Students should check their Uiowa e-mail addresses on a daily basis for communications from the instructor. In addition, e-mail should be used to ask questions or schedule appointments outside of the set office hour times. I will respond to e-mails within 24 hours of receipt excepting on weekends.

Course Organization:
Classes will be carried out using a mixture of lecture and class discussion at key points. Each class will consist of both of these components. At times, students will be able to work either alone or in small groups on a particular problem, which will be followed again by class discussion. We will also make use of student presentations, both small and large-scale at the end of the semester.

In the study of quivers, material will be presented in a manner in which concepts build upon each other throughout the course, meaning attendance is of high importance. If you must miss a class, please let me know, and I will be glad to meet with you outside of class to discuss what you missed. If you are absent, you are still responsible for the material discussed in class and for all of the announcements made in class.

Calculation of Final Grade:
Homework: 20%
Midterm 1: 15% (5% in-class portion, 10% take-home portion)
Midterm 2: 15% (5% in-class portion, 10% take-home portion)
Midterm 3: 15% (5% in-class portion, 10% take-home portion)
Presentation: 20%
Final Exam: 15%

Grading System:
Grades will be assigned on a curve, which will be determined after the final examination. The curve **will not be lower than** the following: 93-100% will guarantee an A, 90-92.9 A-, 87-89.9 B+, 83-86.9 B, 80-82.9 B-, 77-79.9 C+, 73-76.9 C, 70-72.9 C-, 67-69.9 D+, 63-66.9 D, 60-62.9 D-, less than 60 F.

Course Outline:
Unit 1: *Quivers and Representations* August 24 – September 11

- We will discuss the basic definitions of quivers, quiver representations, and maps between quivers with examples.
- We will discuss the various types of subrepresentations, including indecomposable,

projective, and injective representations. Finally, we will then discuss how *all* quiver representations can be viewed using direct sums of these representation types.

Unit 2: *Quivers of Finite Representation Type* September 14 – October 5

- We will discuss precisely which quivers have finitely many representations up to isomorphism. In doing so, we will discuss the construction of the AR-quiver to help us find indecomposable representations of a given quiver and the irreducible maps between them.
- We will discuss Dynkin classification of quivers and prove Gabriel's theorem to allow us to determine precisely when a quiver is of finite representation type.

Unit 3: *Algebras* October 7 – October 23

- We will review the basic theory of rings and modules from abstract algebra so that we can understand the language of algebras.
- We will carefully define algebras and describe many of the familiar number systems such as the integers by their algebraic structures.

Unit 4: *Bound Quiver Algebras* October 26 – November 20

- Using the language of algebras from Unit 3, we will define and describe the path algebras associated to the quivers of Dynkin types described in Unit 2.
- We will define quivers with relations and study their path algebras, comparing the quivers with relations of finite representation type to the quivers found using Gabriel's theorem.

Presentations: November 30 – December 11

Examinations:
There will be three midterm exams and a cumulative final exam. The three midterms will consist of an *in-class exam* and a *take-home exam* following the completion of each of the first three units. In-class exams will consist of short questions and computations based on definitions and known results. Take-home exams will consist of more open-ended proofs and more elaborate computations. The final exam will only consist of a *take-home* component.

Students will be given five days for each midterm exam. During this five-day period, you may choose any 48-hour block to take the exam. You may not work on the exam outside of your chosen 48-hour time block.

Midterm I: (Unit 1)
In-Class: Wednesday, September 16
Take-Home: Handed out on Wednesday, September 16. Due on Monday, September 21.

Midterm II: (Unit 2)
In-Class: Friday, October 9
Take-Home: Handed out on Friday, October 9. Due on Wednesday, October 14.

Midterm III: (Unit 3)
In-Class: Friday, October 30
Take-Home: Handed out on Friday, October 30. Due on Wednesday, November 4.

Final Exam: (Units 1-4)
Take-Home: Handed out on Friday, December 11. Due on Friday, December 18.

Note that midterm exams will not be comprehensive and will cover material on the assigned unit. Earlier material however may be required in order to understand the current material. The final exam *will* be comprehensive.

***For exam and quiz problems, you are expected to show your work. Answers with no explanation as will not receive any credit.*

Make-up exams will be given only under exceptional circumstances (e.g. illness, mandatory religious obligations, certain University activities, or unavoidable circumstances).

Homework:
Homework sets will be posted on ICON once per week excepting exam and presentation weeks. Each homework set will consist of a mixture of computational problems and mathematical statements to prove. Each problem set will be assigned at least four days in advance of the due date.

The homework for this course is designed to help you master your knowledge related to the topics covered during class. As such, you may work on the homework problems with others. However, please be aware that to master the skills needed for this class, practice is required. Be sure to test your knowledge by doing much of the homework on your own.

Late assignments will generally not be accepted. If for some reason you need an extension on any homework assignment, let me know so we can discuss this in advance. Extensions will only be granted after the deadline only in the case of unforeseeable circumstances. The lowest homework score will be dropped at the end of the semester.

Presentations:
At the end of the semester, in lieu of a fourth midterm exam, all students will be responsible for giving a 25 minute presentation on some topic in quiver representations that we have not discussed. Presentations will be scheduled two weeks in advance, and you will be required to meet with me prior to this time to discuss an appropriate topic for your presentation.

Participation:
I highly recommend that students make use of opportunities to ask questions in class or present problems at the board when offered. As with all math classes, the more you engage in class, the more you will likely get out of it. This particularly holds true in an advanced course such as this one.

Other Expectations of Student Performance:
I expect that you will behave with respect to the other students in the class and to me. In particular this means turning off (or silencing) your cell phone. You should not be sending text messages or listening to audio devices during class. Those who are disruptive to the class will be asked to leave.

Resources for Students
You are encouraged to use the following resources in order to help you with both the written and speaking components of this course:

Writing Center: http://www.uiowa.edu/~writingc/
Speaking Center: http://www.uiowa.edu/~rhetoric/centers/speaking.html

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Administrative Home

The College of Liberal Arts and Sciences is the administrative home of this course and governs matters such as the add/drop deadlines, the second-grade-only option, and other related issues. Different colleges may have different policies. Questions may be addressed to 120 Schaeffer Hall, or see the CLAS Academic Policies Handbook at http://clas.uiowa.edu/students/handbook.

Electronic Communication

University policy specifies that students are responsible for all official correspondences sent to their University of Iowa e-mail address (@uiowa.edu). Faculty and students should use this account for correspondences (Operations Manual, III.15.2, k.11).

Accommodations for Disabilities

A student seeking academic accommodations should first register with Student Disability Services and then meet privately with the course instructor to make particular arrangements. See www.uiowa.edu/~sds/ for more information.

Academic Honesty

All CLAS students or students taking classes offered by CLAS have, in essence, agreed to the College's Code of Academic Honesty: "I pledge to do my own academic work and to excel to the best of my abilities, upholding the IOWA Challenge. I promise not to lie about my academic work, to cheat, or to steal the words or ideas of others; nor will I help fellow students to violate the Code of Academic Honesty." Any student committing academic misconduct is reported to the College and placed on disciplinary probation or may be suspended or expelled (CLAS Academic Policies Handbook).

CLAS Final Examination Policies

The final examination schedule for each class is announced by the Registrar generally by the tenth day of classes. Final exams are offered only during the official final examination period. **No exams of any kind are allowed during the last week of classes.** All students should plan on being at the UI through the final examination period. Once the Registrar has announced the date, time, and location of each final exam, the complete schedule will be

published on the Registrar's web site and will be shared with instructors and students. It is the student's responsibility to know the date, time, and place of a final exam.

Making a Suggestion or a Complaint

Students with a suggestion or complaint should first visit with the instructor (and the course supervisor), and then with the departmental DEO. Complaints must be made within six months of the incident (CLAS Academic Policies Handbook).

Understanding Sexual Harassment

Sexual harassment subverts the mission of the University and threatens the well-being of students, faculty, and staff. All members of the UI community have a responsibility to uphold this mission and to contribute to a safe environment that enhances learning. Incidents of sexual harassment should be reported immediately. See the UI Comprehensive Guide on Sexual Harassment for assistance, definitions, and the full University policy.

Reacting Safely to Severe Weather

In severe weather, class members should seek appropriate shelter immediately, leaving the classroom if necessary. The class will continue if possible when the event is over. For more information on Hawk Alert and the siren warning system, visit the Department of Public Safety website