INCORPORATING MEANINGFUL REFLECTION INTO CALCULUS ASSIGNMENTS

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Abstract: At the college level, it can be difficult to engage students outside the classroom through the use of effective homework assignments. Many different types of homework have advantages and disadvantages for preparing students to better understand the course material and to become more effective critical thinkers. I discuss my implementation of online and written homework assignments in two calculus courses at a small liberal arts college, describe five of the written assignments I gave my students to engage them at a deep level, and explain what challenges I faced and how I attempted to resolve these issues.

Keywords: Calculus, online homework, WeBWorK

1 INTRODUCTION

As a new professor, I knew that one critical aspect of my teaching would be assessment through the work I would assign to my students each week outside of class. I wanted to create homework assignments that would challenge my students and give them the opportunity to take what we discussed in class, practice these skills on their own, and ultimately be able to apply their knowledge to solving different types of problems.

With this mindset, I implemented both online and written homework assignments in my classes.

These homework sets were used in my calculus courses during my first semester at a small, liberal arts college. During this semester, I taught Calculus II and Calculus Ia, the first course of a two-semester sequence designed to bridge the gap to calculus for students with weaker mathematical backgrounds and thus included material from both precalculus and differential calculus. The textbook [?] was used as an optional reference source for both courses; however, I did not require students to utilize this textbook. Over the course of the semester, I used both online homework sets through the online homework system WeBWorK as well as written homework assignments I called "reflection homework" that I created myself.

2 ONLINE AND WRITTEN HOMEWORK SETS

In designing homework sets for the Fall 2016 semester, I considered using online and written homework individually at first, as both appeared to have advantages and disadvantages. Overall, online homework sets appeared to work best for giving students shorter computational questions to consider, and written homework sets worked better for asking questions that required students to think more critically about the course material and apply their knowledge in new ways.

Online homework sets allow students to receive immediate feedback on their work, which has been shown to incentivize students to spend more time working on these sets [?, ?, ?]. Instead of having to wait days for the professor or grader to return assignments, students can immediately know if they answered questions correctly. In addition, online homework has been shown to increase student engagement with the mathematics both outside [?, ?] as well as inside [?] the classroom.

Overall, online homework systems have been shown to increase student satisfaction in their courses [?, ?, ?, ?, ?, ?].

Another advantage to using online homework systems is giving students the opportunity to correct mistakes until they are able to complete homework problems successfully. This gives students the opportunity to reflect on their thinking process and avoid becoming discouraged with homework [?]. One final advantage of online homework systems is the ability to give students different variations of the same problem types, allowing them to collaborate on homework assignments while not working on the exact same problems. For all of these reasons, I found online homework attractive for problems in straightforward, computation-focused assignments.

However, other studies have indicated that online homework may show little improvement over written homework for student learning in some ways [?]. In particular, written homework assignments can allow for more in-depth critical thinking and reflection. For instance, Borasi and Rose [?] showed that by keeping a regular journal about their experiences in class, students were able to increase their content knowledge in mathematics courses by being given the opportunity not just to practice computational skills, but also reflecting upon their work in a meaningful manner. In addition, students often encounter problems in communicating their mathematical knowledge [?], which can be described: "I see it, but I can't say it." By writing down their thought processes, students can improve their ability to communicate math effectively.

Green [?] described the writing process in calculus as having two key components: purpose and audience. Green utilized the classification of Meier and Rishel [?] to describe the purpose of writing assignments to be personal, informational, or argumentative. Personal assignments are ones that rely only on students' intuitions or interests and do not necessarily rely on evidence or logic. Informational assignments are ones in which students piece together information from supporting materials. Finally, argumentative assignments are ones in which students must organize and analyze facts to come to a logical conclusion or opinion.

Audience, on the other hand, as used by Meier and Rishel [?] refers to whom students should write. Green [?] determined that if students merely assume that the audience for their assignments is their instructor or grader, they will be less focused in their work and will not restate the problem and thus fail to come to terms with what they are asked to do in assignments. Thus, it is critical to inform students to whom they should be writing their assignments and what they can assume their audience will understand.

Finally, in creating any writing assignment, it is important to ask students good questions. Miller, Santana-Vega, and Terrell described good mathematics questions as ones that will stimulate student interest and curiosity, help students to monitor their understanding, offer students frequent opportunities to make conjectures and argue validity, draw on students' prior understanding, provide instructors a tool for assessment, and support an active learning environment [?]. While not all questions will necessarily address all of these characteristics, it is important to address as many of them as possible to allow for meaningful experiences in completing assignments. In particular, the classification of Miller, Santana-Vega, and Terrell of good questions describes well the different categories of Meier and Rishel [?] of personal, informational, or argumentative assignments.

Ultimately, after considering the benefits and drawbacks of online and written homework assignments, I determined that both types would play a role in my classes. I used the open-source online homework system WeBWorK [?] to assign my students daily online homework assignments that each consisted of a small amount of questions to capitalize on the advantages of online homework of instant feedback, unlimited attempts, and variations in problems for individual students. In order to incorporate assignments that required more personal reflection and critical thinking, I also assigned weekly written homework sets for my students. In keeping with the deeper nature of these writing assignments and to differentiate these homework sets from the online assignments, I called this type of homework "reflection homework."

3 REFLECTION HOMEWORK

Unlike the online homework, written reflection homework assignments were only assigned once per week. Each set consisted of 1-3 questions, though many of these questions had multiple parts. These homework questions can be categorized in the manner of Meier and Rishel [?] as being personal, informational, or argumentative assignments. Some assignments contained questions coming from more than one of these categories. I will now describe several of these assignments taken from my Calculus I and Calculus II courses, categorize them based on the three purposes of Meier and Rishel, and describe their overall effectiveness in enhancing student learning. While some of the reflection assignments I developed during the semester used questions adapted from [?], the ones described below were all designed by me.

3.1 ASSIGNMENT I

Perhaps my favorite reflection homework of the semester was an assignment in Calculus II that preceded formal class discussion of infinite series and which combined all three types of question purposes. In this assignment, I asked my students to watch a Youtube video [?] which "proved" to them that the sum of all natural numbers

$$1+2+3+4+5+\cdots$$

evaluated to $-\frac{1}{12}$ through the use of adding clever multiples of this sum to itself. In this assignment, I first asked students to rewrite the proof in this video in their own words, an informational question. I then asked students the personal/argumentative question of what they believed: did the sum of all natural numbers actually evaluate to $-\frac{1}{12}$, or had the video made a mistake at some point? This question was both argumentative in that I asked students to evaluate an explanation for correctness and was personal in that my students had no real way to formally assess its validity other than their own personal intuition.

This assignment was my favorite of the semester due to the variety of student responses to the second question. Some students accepted the video's claim based only on the authority of the physicists who created the video. Many students found "flaws" in the video's argument that were not actual mistakes. Others did not know what to think and were ultimately torn between accepting the authority of the video's creators and believing their intuition which told them the sum of infinitely many positive numbers could not possibly be a negative number.

For the second question as well as for all other questions of this type where students were asked to come to an uninformed opinion on a mathematical question, grades were not based on the correctness of their solutions but rather based on the depth of their thought processes. In general, as long as students made some sort of argument and showed they had carefully considered the question, they received full points for the question.

I followed up on this assignment later in the semester during a take-home exam when students were asked to again consider this argument in the video after formally discussing series in class for several weeks. At this point, evaluating the proof in the video became less of a personal question and more of an argumentative one in which they were actually able to use tools from the class to show that by the definition of convergence we used, the series above could not possibly converge to any finite number. (In fact, what the video left out was that the series converges in the sense of Ramanujan convergence as seen in [**?**]. At the end of the semester, I told my students there in fact was an alternative definition of convergence that the video used in order to satisfy my students' curiosity.)

3.2 ASSIGNMENT II

One important assignment for my Calculus Ia students was one early in the semester during our discussion of different types of functions, including power, trigonometric, exponential, and rational functions. Students were asked to create a "journal of functions" defined over the real numbers satisfying different properties. I gave students the following categories of functions to consider:

- Functions whose domain cannot be written as a single interval
- Functions whose domain consists of all real numbers
- Functions that are periodic, meaning their behavior repeats itself over time: for all values of x in the domain and for some fixed real number k, $f(x + k) = f(x)$.
- Functions that are increasing over part of their domain and are decreasing over another part of the domain
- Functions that are always decreasing (from left to right)
- Functions that have multiple x-intercepts
- Functions that pass the horizontal line test: any horizontal line intersects the graph of the function at at most one point.

- Functions whose range does not extend to ∞ *and* does not extend to $-\infty$, ie: for which the range lies in some interval of finite length.

For each category, students needed to find two different functions satisfying that property, sketch a graph of each function, then explain why each function satisfies those properties analytically, meaning not just from looking at the graphs. This reflection homework was both informational in that students were required to discern facts about various types of functions and was argumentative in the sense that students had to be able to explain why they picked the functions they chose and why those functions satisfied the desired properties.

This particular reflection homework was unique in that students had three weeks to work on this assignment rather than one week as with all of the other reflection assignments. I assigned this problem set at the beginning of our unit on different types of functions. Throughout this unit, when students encountered different families of functions, they were able to see if those functions fit any of the above categories and keep track of it for this extended assignment.

I was worried for this reflection homework that lots of students would try to pick the same function twice but perhaps translated a few units in order to avoid having to come up with two functions that behaved differently, but in the end this did not happen frequently – while some categories had predictable function choices such as all of the students choosing $\sin(x)$ and $\cos(x)$ for periodic functions, I was also surprised by the complexity of functions that several of my students displayed, for instance looking at combinations of functions such as summing together absolute value and power functions in creative ways.

3.3 ASSIGNMENT III

One reflection assignment that had a very informational purpose was one in which I asked my Calculus I students to explore the limit

$$\lim_{x \to 0^+} \sin(1/x).$$

Rather than giving them this question in a vacuum, however, I instead began the assignment by asking them to use a calculator to evaluate the function $f(x) = \sin(1/x)$ for several different values of x. I gave them several values of x to begin with, including $x = \frac{1}{\pi}$, $x = \frac{2}{\pi}$, $x = 0.1$, $x = 0.01$, and $x = 0$ (which is not in the domain of f), then encouraged students to try several other inputs for x on their own, in the end creating a table of values of the function. Ultimately, students needed to find two sequences of x-values converging to 0 from the right whose values when plugged into the function gave unique values, namely the sequence

$$f\left(\frac{1}{\pi}\right),\ f\left(\frac{1}{2\pi}\right),\ f\left(\frac{1}{3\pi}\right),\ f\left(\frac{1}{4\pi}\right),\ \cdots,$$

which is identically 0 and the sequence

$$f\left(\frac{2}{\pi}\right),\ f\left(\frac{2}{5\pi}\right),\ f\left(\frac{2}{9\pi}\right),\ f\left(\frac{2}{13\pi}\right),\ \cdots,$$

which is identically 1. Using this information then, students were asked to evaluate the limit.

This assignment gave students some degree of creativity in the choices of values they can plug into the function f while still leading them down my desired path of coming to the conclusion that a limit cannot take on two (or more) different values at the same time. I found that while students were able to convincingly explain why the two different sequences converged to different values, they had much more difficulty using this information to determine that $\lim_{x \to 0^+} \sin(1/x)$ does not exist. In particular, many students said that this limit must equal 0 using information just from the first sequence. After talking with a few students, they told me that the first sequence of values more convincingly converges to 0 as opposed to the second sequence of values, which appeared more confusing to them.

3.4 ASSIGNMENT IV

After discussing improper integration in my Calculus II course, I gave students a reflection assignment relating to properties of Gabriel's Horn, the mathematical surface obtained by rotating the area under the graph of the function $f(x) = \frac{1}{x}$ around the x-axis for $x > 1$. I began by asking my students to prove that the surface has finite volume but infinite surface area using improper integrals. I then presented students with the following idea called the painter's paradox in [**?**]: as a three-dimensional surface with finite volume, we should be able to fill the inside of Gabriel's Horn using a finite amount of paint. However, as a surface with infinite surface area, the inside of the horn (which must have the same surface area as the outside of the horn since the horn has zero thickness) cannot be painted using a finite amount of paint.

As a question with a personal purpose, I asked students to think about how this paradox could possibly be resolved. Just like the question from Assignment I, I told students that their answers would not be evaluated on correctness but rather based on showing that they thought about this paradox and attempted to try to explain it in some way. Overall, I was impressed that several students came up with ideas that, if properly formalized, would describe ways in which this paradox could be resolved using only a finite amount of paint. Other students, who were still very confused after thinking about this problem, mentioned in their write-ups that they enjoyed talking with each other and pondering the solution to this problem with one group even attempting to construct a cone to represent Gabriel's Horn and fill it with water to test their hypothesis.

3.5 ASSIGNMENT V

One of the earliest reflection assignments that I gave to my Calculus II class was one that asked students to think about the concepts of codomain and range of a function and one-to-one functions in a context different than what they had seen in class by creating examples of functions without numerical inputs or outputs that satisfied different properties relating to these concepts. For instance, one question asked students to describe a function $f(x)$ satisfying all three of the following properties:

- The domain of $f(x)$ is the set of novels that have ever been written.
- $f(x)$ is *not* a one-to-one function.
- The range of $f(x)$ is *not* equal to its codomain.

Students were required to define the domain, range, and codomain of $f(x)$, a description of what $f(x)$ does to an input, and explanations as to why $f(x)$ satisfies each of these properties. I then asked students several similar questions in which they needed to generalize the notion of one-to-oneness beyond just describing functions that pass the horizontal line test. This assignment was designed as an argumentative one in which students not only had to understand these concepts we had discussed in class but also generalize them to a much broader class of functions than those typically studied in a calculus course.

4 IMPLEMENTATION OF REFLECTION ASSIGNMENTS

In creating all of my reflection assignments, I emphasized one key aspect that I believe could be added to Miller, Santana-Vega, and Terrell's characteristics of good questions: giving students a way to express creativity in mathematics. I believe that students often have a pre-conceived notion that calculus is merely a process of following the correct formulas in the correct order and does not have room for creativity in exploring its applications. Through creating meaningful reflection assignments, I attempted to give students the opportunity to explore questions in different ways, come up with new ideas, and creatively engage with the course material in order to help students develop a new outlook on mathematics.

I did not want to create assignments, however, that were too open-ended or creativity-based, as I worried that students at the calculus level would not respond well to assignments without any questions for which they could be objectively sure of their answers. Thus, in creating questions for which I solicited student opinions based on intuition alone as in Assignments I and IV, I also included questions such as asking students to restate arguments in their own words or calculating volume and surface area using techniques from class. This had the added bonus of giving both a means to assess how well students understood previous course concepts as well as a way to give students the opportunity to come up with new ideas without the fear of being penalized for being factually incorrect.

After each reflection homework was graded and passed back to students, I posted solutions for that assignment to the course website. Sometimes, my solutions were absent or deliberately vague, which usually meant I planned to ask students the corresponding questions again later in the semester. Typically, I wrote out complete solutions for most reflection assignments. At the end of the semester, my students reported that they enjoyed reading my explanations for the intuition-driven questions, as those were often the ones they were most curious about after each assignment. They also found my solutions to be cumulatively helpful as a model for how to write complete mathematical explanations.

Because the homework for my class came in the form of both straightforward computation in online homework and more open-ended questions in reflection homework, I designed assessments for my courses to test my students' abilities in both of these areas. In each course, students were given weekly quizzes that asked them to complete computations and short answer questions. Both of the two midterm exams for each class had in-class and take-home components that were weighted equally for grades. The in-class exams were structured much like the quizzes in that they asked computational and short-answer questions, while the take-home exams each asked 1-3 questions that were similar in nature to questions from the reflection homework. In a few instances, the take-home exams referenced questions from reflection assignments directly such as asking students to re-evaluate the proof that the natural numbers sum to $-\frac{1}{12}$ after we discussed series formally as a class. In this way, I structured major assessments for my class to be similar to both the online and written homework assignments.

Overall, students reported feeling more confident in their performance on the in-class than on the take-home exams in their semester evaluations. However, several also acknowledged that they were used to taking in-class exams in math and that the notion of take-home exams made them more nervous. They overall seemed to appreciate being given the opportunity to show their understanding in two very different types of major assessments. I found that while students felt more confident in their performance on in-class than on take-home exams, their actual performance was comparable on both. While many students performed significantly more strongly on one type of assessment than the other, the class averages on these exam types were very similar, leading me to believe that students felt more uncomfortable with take-home assessments due to their lack of experience with these questions from prior math classes. To help combat this, in the future I plan to use more in-class writing activities where students can read and critique each other's work in order to help them build more confidence in their writing throughout the semester.

I enjoyed creating reflection homework sets and found that the students who talked to me about these problems enjoyed working on them, especially the more open-ended problems. One common issue I encountered with reflection assignments, especially earlier in the semester, was that students were often confused by my instructions and would have to ask me for clarification. I frequently sent out clarification e-mails to the whole class and found it a personal challenge to write questions in a way that made sense to students. In my evaluations at the end of the semester, a few students said that reflection assignments did not correspond very well to what they had discussed in class and did not seem to be very useful for them while learning topics for the first time. This comment in particular was more prevalent from my Calculus Ia students than from my Calculus II students. Finally, several students commented that weekly reflection assignments in addition to daily online homework sets caused too much time to be spent working on their calculus classes. In particular, they thought that forcing a reflection assignment to be due each week got to be somewhat tedious later in the semester.

5 CONCLUDING REMARKS

Overall, I found reflection assignments to be successful in their purpose. After my first semester of assigning reflection homework sets, I continued using them in both of my calculus courses the following semester. However, instead of making these assignments due weekly, I assigned slightly longer homework sets to be due once every two weeks. This resulted in assigning these sets at a rate which allows students to spend more time working on individual assignments without feeling rushed. This change also relieved pressure to try and find meaningful questions to ask for each assignment every week. I found that as time went on, I became more skilled at writing assignments and did not need to send clarification e-mails to my classes as frequently. I have ultimately found that while creating reflection assignments that are both challenging and engaging can be a very challenging task, most students appreciate being given the opportunity to explain their thought process and think about math in new ways that allow them to view mathematics as a more creative science.

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BIOGRAPHICAL SKETCHES

Author received his undergraduate education at a liberal arts college in Ohio and is interested in finding ways to better engage students in the collegiate environment. He likes to read books, hike, and travel with his family.