MTH 320: Midterm 2

March 28, 2019

Name: ____

Make sure to **show your work or explain your answers** for all (non-T/F) questions; I reserve the right to give no credit to solutions with no work or explanation. Good luck, and you got this!

1. (2 points each) Indicate whether the following statements are true (T) or false (F) by circling the correct choice. If a statement is correct only some, but not all, of the time, you should choose false (F). You do not need to explain your answer.

T / F a) If $f(x, y, z) = \cos(z)$, then $\nabla f(x, y, z) = -\sin(z)$.

T / F b) For any C^1 function $f : \mathbb{R}^3 \to \mathbb{R}$ with $\mathbf{a}$ in $\mathbb{R}^3$, $D_{(1,1,0)}f(\mathbf{a}) = \frac{\partial f}{\partial x}(\mathbf{a}) + \frac{\partial f}{\partial y}(\mathbf{a})$.

T / F c) The minimum value of the directional derivative $D_{\mathbf{u}}f(\mathbf{a})$ for a fixed $\mathbf{a}$ is given by the expression $|| - \nabla f(\mathbf{a})||$.

T / F d) Let S be the surface defined by the set of points satisfying $f(x, y, z) = 6$ for some continuous function f. If $\mathbf{a}$ is a point on S satisfying $\nabla f(\mathbf{a}) = (0, 0, 0)$, then S does *not* have a tangent plane at $\mathbf{a}$.

T / F e) The range of the function $f(x, y) = \left(\dfrac{2}{\sqrt{x^2 + y^2}}, x^2 + y^2 \right)$ is the set $\{(u, v) \text{ in } \mathbb{R}^2 : \ u > 0, \ v \geq 0\}$.

2. Consider the function $f(x, y) = |y|$.

a) (4 points) Sketch the level curves of f at heights $c = -2$, $c = 0$, $c = 1$, and $c = 2$.

b) (4 points) Using part a) or otherwise, sketch a graph of $z = f(x, y)$.

3. (4 points) Find formulas for the component functions of the function $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$f(\mathbf{x}) = (6\mathbf{i} - 5\mathbf{k}) \times \mathbf{x}.$$

4. (4 points) Find an equation for the plane tangent to the graph of

$$f(x, y) = x^3 - 7xy + e^y + 2$$

at the point $(-1, 0, 2)$.

5. (7 points) Find *all* critical points of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = x^2 - \frac{4}{3}y^3 - x^2y + y.$$

Determine if each critical point is the location of a local maximum, local minimum, or saddle point.

6. (6 points) Suppose that Epona the flying horse is running along the curve

$$g(t) = (2t, t^3, e^t)$$

in $\mathbb{R}^3$ when Epona encounters a weather front so that the barometric pressure is varying wildly from point to point as

$$P(x, y, z) = x^2yz \text{ atmospheres.}$$

Use a version of the *multivariable* chain rule to determine the rate of change of the pressure Epona is experiencing at $t = 1$ minutes.

7. (6 points) The surface of the Great Plateau can be represented by a region in $\mathbb{R}^2$ where the temperature at any point (x, y) is given by the expression $T(x, y) = x^2y^3 - 50\ \ ^\circ C$.

If Link is standing at the point $(2, -1)$, what direction should Link walk in so that the temperature will decrease the most rapidly? Give your answer as a unit vector.

8. (5 points) Evaluate the limit, or explain why the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{x^6+y^6}$$

BONUS (1 point): Describe *two* things that you can do that would make another person happier this weekend. Now do them!