Worksheet 10
April 19, 2014

Using Trigonometric Identities

Here are the trigonometric identities that we have learned in the last couple of lessons. Note that these will be given to you on any future quiz or exam that requires their usage:

Pythagorean Identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $1 + \cot^2(x) = \csc^2(x)$

Sum + Difference Identities

- $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

Double-Angle Identities

- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$

Half-Angle Identities

- $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos(x)}{2}$
- $\cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos(x)}{2}$

The difficult part about using these identities is knowing when to apply which ones to get useful results. This is made even trickier by the fact that sometimes you can use different rules to get the same results.

EX: If we trying to find $\cos\left(\frac{7\pi}{12}\right)$, we can use that
$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ and thus get our result using a sum identity.
We could also use that $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\frac{7\pi}{6}}{2}\right)$ and thus get our result using a half-angle identity.

We also have to be careful in that since lots of square roots will be taken in using these identities, it is important to know whether for example $\sin(x)$ or $\cos(x)$ should be positive or negative.

EX: For $x \in [0, 2\pi]$, suppose that we know $\tan(x) < 0, \cos(x) > 0$. This means $P(x)$ must be in Quadrant IV, implying that $\sin(x) < 0$ here. We also know then that $\frac{3\pi}{2} < x < 2\pi$, implying $\frac{3\pi}{4} < \frac{x}{2} < \pi$, meaning for instance that $\cos\left(\frac{x}{2}\right) < 0$ and $\sin\left(\frac{x}{2}\right) > 0$, as $P(x)$ lies in Quadrant II for any x between $\frac{3\pi}{4}$ and π.

Try these on for size:

EX: If $\sin(t) = -\frac{4}{5}$ and $\pi < t < \frac{3\pi}{2}$, find

a) $\sin(2t)$

b) $\cos(\frac{t}{2})$

EX: Find the following values/points:

a) $\sin(-\frac{5\pi}{12})$

b) $\tan(\frac{11\pi}{8})$

EX: Find all $t \in [0, 2\pi]$ solving the following equations:

a) $\sin(2t) = \cos(t)$

b) $2\cot^2(t) + \csc^2(t) - 2 = 0$